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ABSTRACT

This study sought to describe and identify trends in teachers' affect toward, conceptions of, and understanding of mathematics as they proceeded through 30 hours of collegiate mathematics to receive a second certificate to teach secondary school mathematics. From the 24 teachers who began the program, 13 teachers were selected because they were currently teaching mathematics; complete case studies were conducted of seven of these teachers. This paper presents three of the seven case studies. The methodology is described, followed by discussion of theoretical bases for affect, conceptions, and understanding. The three case studies are presented, with details pertaining to the three categories. Several similarities among the teachers indicated that the program fell short of adequately preparing these teachers for mathematics teaching. Course descriptions and schedules are included. (MNS)

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Teachers Seeking a Second Certificate in Mathematics: Affect, Conceptions and Understanding

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Teachers Seeking a Second Certificate in Mathematics: Affect, Conceptions and Understanding

Current shortages of qualified secondary mathematics teachers have created a new pool from which to draw qualified teachers--experienced teachers from other disciplines and with little preparation in mathematics. With few teaching jobs in their own fields, teachers who desire to remain in teaching are returning to school to obtain sufficient coursework to become certified to teach secondary mathematics. To meet the needs of both the profession and these teachers, programs to certify experienced teachers have blossomed across the country. However, these teachers present a dilemma for teacher educators. While the teachers often bring to the programs bountiful experience in the classroom, they also bring with them a weak mathematics background. Often mathematics was not their first, or even their second, choice of teaching fields. Therefore, their previous study of mathematics has been minimal. To date, no research focusing on these programs or their participants has been conducted. Little is known about the effects of such programs on the teachers seeking a second certificate.

Purpose

Specifically, this study sought to describe and identify trends in teachers' affect toward, conceptions of,

and understanding of mathematics as they proceeded through 30 semester hours of collegiate mathematics to receive a second certificate to teach secondary mathematics. On a more global level, this study provided an opportunity to describe the effects of a traditional sequence of collegiate mathematics courses on teachers' conceptions, affect and understanding.

The Certification Program

The study was conducted in the context of a program to certify elementary, junior high and high school teachers of other disciplines in secondary mathematics. A small Texas university was awarded $140,000 to provide a program to alleviate a shortage of qualified mathematics teachers in the area. Teachers in the program were required to complete 30 semester hours (10 courses) of collegiate mathematics. See Table 1 for a list and brief description of each course.

Insert Table 1 about here

The program began in the summer of 1984 and was scheduled to end in the summer of 1985. However, due to the fast paced, highly concentrated nature of the program, many teachers finished the program at a later date. Of the 24 teachers who began the program, five finished in the summer of 1985, two finished in the fall of 1985, three finished in

the spring of 1986, four finished in the summer of 1986, and 10 withdrew from the program.

Subject Selection

From the 24 teachers who began the program, 13 teachers were selected because they were currently teaching mathematics at either the elementary, junior high, or secondary level. Six of these 13 teachers withdrew from the program; therefore, complete case studies were conducted on seven teachers. This paper presents three of the seven case studies.

Conducting the Case Studies

The methodology of this study was the multicase study. The teachers of the study were visited in their school on three or four occasions over a two-year period. During each visit, one of the investigators observed, audiorecorded and took field notes for each mathematics class taught. Additionally, the teachers completed several written instruments and were interviewed with respect to their affect, conceptions and understanding. Although, the interviews were structured, the investigators often deviated from the script to pursue promising and revealing remarks. Each teacher kept a weekly log of their thoughts and feelings during the summer and fall of 1984. A schedule of the visits and coursework of the three teachers can be found in Table 2.

Theoretical Bases for Affect, Conceptions and Understanding

Affect was defined as "a wide range of concepts and phenomena including feelings, emotions, moods, motivation and certain drives and instincts" (Corsini, 1984, p. 32). For this study, affect toward mathematics focused on such feelings and emotions as enjoyment of mathematics, appreciation of mathematics, anxiety toward mathematics, and confidence and frustration in teaching and learning mathematics. The first two were measured through Aiken's (1974) E-Scale and V-Scale. Anxiety was measured by the Mathematics Anxiety Rating Scale (MARS) (Richardson and Suinn, 1972). Interview questions concentrated specifically on these and the other affective factors.

Conception of mathematics was based on a developmental framework of conceptions of knowledge developed by Perry (1970). He asserted that beliefs about knowledge evolve through four major stages: dualism--any proposition or act must be right or wrong, multiplicity--a plurality of viewpoints exist, but no internal structure or external relationships exist, relativism--a plurality of viewpoints exist, context is very important, and commitment--one personally commits to a mode of action. Teachers' positions in Perry's framework were ascertained through responses on investigator-designed written instruments and in interviews.

Skemp's (1978) concepts of relational and instrumental understanding were used to analyze and describe the mathematics understanding of the teachers. Skemp defined relational understanding as knowing what to do and why you are doing it and instrumental understanding as knowing only what to do. Because the subjects of the study were, at the same time, both learners and teachers, we sought to investigate their understanding with respect to both learning goals and teaching goals. Specifically, were they learning mathematics instrumentally or relationally in the certification program? Were they teaching mathematics instrumentally or relationally in their classes? These questions were answered through responses to written instruments and interview questions and analysis of teaching transcripts and field notes.

Three Case Studies

Upon analysis of the written instruments, interview transcripts, lesson transcripts and coding, and weekly logs, we attempted to describe affect, conceptions and understanding of the three teachers over the two years of the program. Table 2 presents a summary of the coursework taken by each teacher and our classification of her orientation at each interview time. The case studies which follow are offered to provide evidence and credence for our classification. We have structured the case studies

according to the constructs of affect, conceptions and understanding to highlight changes, or lack thereof.

Pam

Pam was teaching ninth-grade Algebra and tenth-grade Geometry at a local Catholic high school prior to starting the certification program. She had nine years of elementary school teaching experience, six of those years in the first grade, and two years experience as an elementary school librarian. She obtained an elementary teaching certificate in 1964 and an all-level librarian certificate in 1980. Her primary motivation for enrolling in the certification program was to retain her present teaching position. "If it had been any other job, I would have chosen something else." Luckily for Pam, however, mathematics was her favorite subject. "I've always liked mathematics. That was my best subject all through elementary, junior high, high school, and college. It's the one that came the easiest." The timing of the program was good as Pam had already completed three required courses: Trigonometry, Analytic Geometry, and the methods course.

Pam elected to retake College Algebra, Trigonometry and Analytic Geometry during the first summer of the program. In Fall 1984, she enrolled in Calculus I, but eventually withdrew with twelve other teachers because of heavy teaching responsibilities and problems with the instructor. In the fall, her teaching responsibility changed. She moved

from the private Catholic school to a public junior high school where she taught seventh- and eighth-grade general mathematics. She completed Calculus I the following spring and completed four courses in Summer 1985. She finished the program one semester late by completing Linear Algebra in Fall 1985.

Affect. Prior to the certification program, Pam's attitude toward mathematics was generally positive. "I like it. It's something fun to do. It's something that stimulates something in me." However, some aspects of mathematics made her uncomfortable. "The proofs I don't enjoy that much. I don't like to do word problems either." Much of this distaste was the result of previous instruction. "We didn't do many word problems. When we did, there never seemed like there was a clear cut way to go about it. Whoever discovered the way to do it didn't tell us." As long as precise procedures existed, she was comfortable. When the mathematics was more open-ended, she was uncomfortable. While she claimed no general anxiety toward mathematics, she admitted some anxiety prior to tests. Her responses to items referring to tests on the MARS supported her remarks. Her confidence with algorithms, procedures and concepts was high; her confidence with mathematics requiring high-level thinking was low. This lack of confidence was best illustrated by her choice to retake the courses during the first summer.

In the spring of 1985, she was still fond of mathematics. "Yeah, I still love it." However, several of the courses had an impact on her attitude toward teaching. "I've come across some instructors I decided I would not be like. That has changed my way of adding more humor in the math I teach and try to make it more exciting." Her math anxiety increased slightly, primarily due to the stress of testing situations in her coursework. In her diary, she expressed some frustration in Probability and the first Calculus I course. To her, the instructors spent far too much time on theory and not enough time on applications. She was frustrated that she could not grasp the theory.

In the fall of 1985, Pam's attitude toward mathematics remained high. "I still enjoy it. I wish I understood more, but I still like it. Math is challenging, stimulating and, many times, lots of fun. It's like a puzzle; when the pieces fit, it provides great satisfaction. When they do not, there is a challenge of turning and manipulating until it does." Her math anxiety remained higher than it was at the outset, due primarily to the coursework. "The harder the mathematics, the more abstract, the more theoretical, the anxiety increases."

Conception. Pam entered the program with a relativistic view of mathematics. She felt that the truth of mathematical rules and facts were based on context and that mathematics was subject to interpretation. She clearly

saw the dichotomy of college mathematics and school mathematics and recognized that most of the mathematics taught in schools were driven by single correct procedures and answers.

In the Spring 1985 interview, Pam's conception of mathematics remained relativistic, and her view caused problems in her teaching. "Sometimes I get a little distressed because I go in at a different method than the book does. Sometimes I get a little anxious about that because I don't want them to learn the wrong method. Even though there's many ways to do it, I get a little anxious sometimes when I teach the wrong way." Her lack of confidence surfaced again. She was not at all comfortable in her relativistic world; her comfort lay in a dualistic world. "I enjoy basic math because there is usually a definite answer for a specific question." She recognized that mathematics is dependent on context and subject to interpretation; however, she preferred a dualistic mathematics.

Pam's conception of mathematics remained relativistic through the final interview, and her view was constantly verified by the coursework. "With each course I've taken, I've seen fewer definite methods of solution and more interpretive methods. It seems the higher the mathematics, the more methods for solution are available. I began to see that there is so much I didn't really know was there. I

thought it was a smaller field." While her relativism was supported, she remained more comfortable in a dualistic environment. "What I like most about mathematics is that you usually can get an exact answer or you can look in the back of the book and see if you got it right." Pam clearly held a relativistic view of mathematics throughout the program; however, evidence suggested she preferred dualism.

Understanding. Prior to the program, Pam admitted that her learning of mathematics had been highly instrumental. Past teachers had never "explained the process." She believed that mathematics was best learned through memorization and practice. Her explanations of finding the area of a parallelogram, why division by zero is impossible, and why the empty set is a subset of all sets were based on rigid rules or accepted as fact. Her teaching matched her belief and had a definite instrumental slant. In her Algebra lesson after giving the rule for multiplying fractions, she told the class, "If you would like to know why, you can read the proof in the book. If you accept it like this, then that is all right." Explanations and reasons why were not a high priority in her teaching.

In the spring of 1985, little evidence of changes in understanding surfaced. Her explanations of why zero is neither even nor odd, of why sin x/cos x = tan x, and of changing a decimal to a percent were highly instrumental, i.e. sin x/cos x = tan x "because the textbook says so."

She readily admitted that some persons just "never seem to understand the why in math. For them, memorization and practice is their only course." One entry in her diary indicated that she may have been one of those persons and was undergoing a change. "The straight line and conic sections were a review of previous work, but an interesting thing occurred. Instead of memorizing formulas, I began to understand the parts of the formulas and how those parts affected the graphs." Her study strategy represented a change in the previous study pattern, "through repeated usage of a theorem or definition I begin to understand it." While her learning appeared to be moving in a relational direction, no evidence of relational teaching was found. She continued to emphasize procedures and avoid reasons--now in seventh- and eighth-grade general mathematics classes.

During her final interview in the fall of 1985, she admitted that her progress through the coursework of the program had been "more of a memorization process than a learning process." The primary reason was time. With the pace of the coursework, she simply did not have time to understand the material to her satisfaction. She expressed a desire to learn relationally, but time would not permit it. Pam's teaching, however, appeared more relational. She was giving reasons for procedures and attempting to justify them. Perhaps the move from algebra and geometry to junior high mathematics facilitated the change. After all, Pam was

more comfortable in a dualistic world. Her comfort and a better understanding of the subject manner may have allowed her to teach in a manner in which she preferred. She never did like proofs and word problems and had difficulty understanding them.

Tina

Tina graduated from a small university in 1973 with an all-level certificate to teach art. After an "eye-opening" year teaching art and crafts in a school for emotionally and physically handicapped, she decided to leave teaching. She moved away from home and in five years had worked her way up to management level in a printing company. She became disillusioned, sought a teaching position and found one in her hometown. All was well until the school district began eliminating art from the secondary curriculum. She was assigned two general math classes and "encouraged" to obtain a teaching certificate in mathematics. The timing of the certification program was perfect for Tina. It represented a means by which she could stay in teaching--she entered for survival. She had just recently completed three courses--College Algebra, Trigonometry, Analytic Geometry--on her own. Her performance in the latter was so poor that she was required to repeat it.

During the first summer term, she completed Analytic Geometry and Probability with a B in each. In the fall, she was one of only six students who remained in Calculus I.

She received an A in the course. Also in the fall, she was assigned to teach Honors Algebra I in addition to two general math classes during the 1984-85 school year. In the spring, she completed Calculus II with a B and the methods course with an A. During the summer of 1985, she finished the program by completing the final four courses. Only five teachers completed the program on time. Tina was one of them, and she was quite proud of it.

Affect. Tina's attitude toward mathematics prior to the program could be classified as neutral at best. Mathematics ranked a distant third behind art and history as a love. "I'm not very emotional about math. Art I get excited about. Math is just kind of there." She was confident in her mathematical ability--especially in computing and estimating. She had done well in accelerated mathematics classes in high school. However, she lacked confidence in her mathematics teaching. "Being an art major, teaching math does not give me a feeling of security." While she was not particularly anxious about mathematics, she was concerned about her ability to succeed in the program. Her recent difficulty in Analytic Geometry had shaken her confidence.

The Fall 1984 interview revealed several changes in affect. High grades in the initial coursework caused Tina's confidence to rise. "I've been getting good grades in my work, and that's reassuring. It makes you feel good." Her

confidence in teaching mathematics also rose. "I feel knowing more math does help me teach math in a better way." With success and confidence also came a more positive attitude toward mathematics. "I like math. I really do. More so in calculus. I can see the artistic angles in it, and I enjoy that. It feels like a real accomplishment. It's fun." However, not all was positive. Math anxiety increased due to the intensity and pace of the courses. "I have to be good at it or I get frustrated. This summer I really got upset a couple of times. It was just a lot of new knowledge real fast, and very frustrating being pushed." Upcoming coursework remained a concern. "I'm concerned how I will function [in Calculus II] next semester because I could not work the problems [in Calculus I] without the book in my hand."

Tina's attitude toward mathematics remained positive in the spring interview. "It's more fun to be challenged like this instead of fooling around with the same things you've done in the fifth grade." Her anxiety toward mathematics was isolated to testing situations. "It doesn't scare me much anymore. Most of the time, when I'm working problems at home, there's not so much anxiety because I've got the book. But test time was a time of high anxiety. I hit the panic button." Her confidence in teaching remained very high. The coursework had given her "a better idea of the whole big picture of mathematics, a deeper understanding of

mathematics, insight as to how to set up a lesson format, and assistance in responding to classroom questions."

In the final interview, Tina's attitude was at a high, and her anxiety was at a low. "I feel much more confident about it. I'm not scared of anything anymore." The pressure was finally off. "I had a great desire to be an A student the entire time. By the end of 27 hours [of coursework], I was not caring if it was an A or a C." What she had enjoyed most about mathematics was its challenge. What she liked least about mathematics was its frustration. However, through much frustration and work she had conquered the program. But the conquest was not without a casuality. "Teaching full time and taking 27 hours of math courses at night wore me out. I'm kind of numb right now. I'm just flat numb--about the world and everything."

Conception. Tina's conception of mathematics prior to the program could best be classified as dualistic. She viewed mathematics as a droll, stagnant activity involving no creativity. "When I think of mathematics, I think about numbers. Just numbers." Her comparison of history and mathematics was most illuminating. "History can be interesting and creative. There is more creativity in history than in math. It's just memorization to an extent. There is not much expansion to it. There is no blossoming. There is no movement to new ideas."

In the Fall 1984 interview, her conception of mathematics remained dualistic. She viewed mathematics as a game. "Math is like a game, because in a game you have to know all the rules. You have to move all the right pieces at the right time. It's a little bit of luck sometimes. Then if you pick right, you win. And if you pick wrong, you have to redo." Her win-lose concept of mathematics was highly dualistic. Her reference to luck reflected her belief that some entity outside her influenced the outcome.

Tina's conception remained dualistic in both the Spring and Fall 1985 interviews. In the final interview, she remarked, "Mathematics is not like a poem or a painting and subject to interpretation. It is based on unchangeable facts related to the real world. It's numbers and symbols. That's all it comes down to." She also carried her game analogy a step further. "Mathematics is like a game. Today one may relate it to the video games that go to levels of complexity. Level one is the basic operations. Level two introduces fractions, integers and their operations. Level three introduces radicals and exponents. Level four is algebra and problem solving. Trig and geometry are level five. Calculus takes them all and draws the big picture." The coursework had broadened her view of mathematics and shown her complexity; however, it did not alter her dualist view of mathematics.

Understanding. Tina's understanding of mathematics was difficult to ascertain. While she espoused the importance of understanding in order to apply or do mathematics, little evidence of a personal relational understanding was evident. In fact, at the end of the program, she admitted, "When I started taking this stuff, I didn't remember ten terminologies referring to mathematics." At the outset, she could not explain why division by zero is impossible and did not know that a square is a rectangle. Her teaching was highly instrumental. The textbook was viewed as the absolute authority in her general math classes. Her explanations involved citing rules and procedures. Following the textbook's rules and procedures were important in her teaching. Form and format in working exercises were also high priorities.

In the fall of 1984, her personal understanding of mathematics still appeared instrumental. Textbooks and memorization played key roles in her learning of the content of the summer coursework. "I don't have the formulas memorized. It's definitely hunt and peck and searching work. But I can get it done; it's just a matter of having the book open there." When asked why a number to the zero power is one, she replied, "It's just a learned rule." She seemed to have learned mathematics instrumentally and received good grades doing so. Furthermore, she seemed to continue her instrumental goal in the certification

coursework. Her teaching also remained instrumental. Memorization of rules and procedures remained prominent. Her Honors Algebra students were required to solve equations using exactly the same procedures as outlined in the textbook. When a student asked why division by zero was impossible, she replied, "You can't divide by zero. It is against all the rules in mathematics to divide by zero."

Although Calculus I and II gave Tina "a better idea of the whole big picture of mathematics," her learning remained instrumental as of the Spring 1985 interview. In discussing her difficulty with tests, she remarked, "You have a lot to remember for a test. I guess that's one of my weakest points. I just can't seem to remember all that stuff. You start taking away my reference materials and I start getting confused on what sine and cosine are." Obviously, Tina had become heavily dependent upon memory and the textbooks to get through her courses. She had not yet developed even a basic understanding of trigonometric concepts. Her goal was to obtain good grades, and it was really taxing her memory. Although her explanations of mathematical laws and procedures during interviews were more relational, her teaching remained highly instrumental. Memorization of rules and procedures and form continued to be important.

Tina's learning remained instrumental until the end. When asked to explain why $\sin x + \cos x = 1$, she replied, "I have never been one to question such material and have not

had a professor that volunteered the information." She continued to espouse the need to develop understanding. "Mathematics is a building process. If understanding is not there, one is not able to think through problems." However, her own personal learning throughout the program was based on memorization and highly dependent upon textbooks and references. Although her teaching appeared to remain predominantly instrumental, evidence of relational teaching did surface occasionally. For example, she explained an example of the distributive property, $5(x+y)$ as $x+y$ summed five times. While it was far from relational, her teaching included richer explanations and more deviations from the textbook than prior to the certification program.

Fran

Fran, a mother of two in her late thirties, was teaching mathematics and science in grades 4 and 5 in a small rural elementary school when she decided to enroll in the certification program. Her undergraduate education provided her with secondary teaching certificates in home economics and history. Her initial teaching experiences included high school history, junior high language arts and homemaking, and secondary special education. Her transfer to an elementary school prompted her to return to school and obtain a Masters and teaching certificate in elementary education. She had been teaching at the rural elementary school for the past seven years.

Fran enrolled in and completed the full schedule of three courses during the summer of 1984. In the fall, she was transferred from the elementary school to a rural junior high school where she taught seventh- and eighth-grade mathematics. She also withdrew from Calculus I with 15 other teachers in the fall because of increasing teaching demands and poor instruction. The following spring she re-enrolled in Calculus I and took the methods course. She successfully completed her spring coursework, and enrolled in three courses the following summer. She was enrolled in her final course Linear Algebra when the final interview was conducted.

Affect. Fran enrolled in the certification program because she enjoyed mathematics. She had enjoyed it as a student. "It's exciting to work with it, and I always get a sense of accomplishment when I make it work right." She enjoyed teaching it. "I would like the children to enjoy it as much as I do. It's exciting to find new and exciting ways to present math and to watch the little light bulbs going on. To watch them really get excited about math." Fran was confident that she would succeed in the program. "I don't give up; I stay with it until I get it." While she claimed to have little fear or anxiety toward mathematics per se, she did express concern for "her own silly fear of inadequacy." "I want to know what I'm doing. I don't want

to feel stupid. I just hope I make it. I don't take being a failure very well."

Despite challenges in the summer mathematics courses and the setback in Calculus I, Fran's attitude toward mathematics remained positive in the Spring 1985 interview. "I like it. I like the satisfaction I get when I complete a problem. When I really understand something, I really feel good." The coursework also helped build confidence in teaching. "I feel like the courses have helped me be more confident about what I know." However, with satisfaction also came frustration when she did not understand. "I'm frustrated about the probability. If I learn nothing else, I will at least be aware of how and why a student can just quit because he/she sees no point in continuing." Fran also had become concerned over her inadequate background and understanding in mathematics, and this concern had caused her level of math anxiety to increase.

In the fall of 1985, Fran's attitude toward mathematics in general had improved--despite earlier frustration. "I enjoy math more. I like it, and I've always liked it. It's a challenge and it's stimulating. I don't feel any differently than I did before." While she claimed no anxiety in general, she confessed to anxiety toward certain courses and topics. "I have to get into a course and be taking it for a while, see what the instructor is like before I'm anxious about it." As a result, some frustration

continued to linger. "Mathematics becomes frustrating only when I can't understand a concept because I lack the proper foundation and can't find the information necessary to solve my dilemma." The good obviously outweighed the bad as Fran continued to enjoy mathematics.

The final interview in the spring of 1986 revealed no new information with respect to Fran's affect. She left the program with a highly positive attitude toward mathematics--just as she had entered.

Conception. Fran's conception of mathematics could best be classified as multiplistic upon entering the program. While recognizing that mathematics problems have multiple approaches and solutions, she viewed mathematics as a precise subject in which facts and rules are either true or false. She saw little room for subjectivity or creativity.

In the spring of 1985, her conception appeared unchanged. She retained her multiplistic view that there were many approaches in doing mathematics; yet, she still viewed mathematics as a precise set of rules and facts. "The more I learn about different types of mathematical content, the more I become aware of varied and different approaches that can be taken toward the solution of a problem." The coursework had reinforced her multiplistic view.

The final two interviews revealed nothing new. Fran was highly consistent in her responses on written instruments and interview questions with respect to her view of mathematics. She entered the program multiplistic and left the program multiplistic. If anything, the coursework merely reinforced her beliefs.

Understanding. Fran's understanding of mathematics prior to the program represented an interesting dichotomy. Her past learning of mathematics appeared to have been highly instrumental; however, her teaching was relational. The portion of the interview requiring her to give mathematical explanations clearly illustrated the differences. Her explanations to help the interviewer understand were clearly instrumental--multiply length times width (sic) to find the area of a parallelogram and move the decimal point to the right to change decimals to percent. However, when asked how she would explain the same procedures to students, her explanations were clearly relational--place a grid of squares on a parallelogram to find the area and consider percent as a fraction with denominator of 100. She firmly believed that "if a child is taught methods and not understanding, there will be little if any transference or application of knowledge from lower to higher levels of learning." Fran was not comfortable with her own personal understanding of mathematics. "My ACT scores were not terrific which led me to believe that if I

was making an A for that teacher, then he wasn't giving me what I needed to know." She admitted that much of her previous learning in mathematics had been memorization. She also referred to the difference in her learning and teaching. "Taking a course and teaching a course is two different things. You would have to have a year or two experience before you could really do a good job of saying most of what children need to have, one step after the other or showing alternative methods." In other words, merely learning the content was not sufficient to teach it properly.

Fran's past instrumental learning and poor background in mathematics became exceedingly apparent during the summer and fall coursework. In her weekly diary, she constantly referred to difficulties in learning the material because of her background. She remarked often that she had no previous knowledge on which to attach the new information. She often relied on other textbooks to supplement the class text and help her understand concepts and terminology. Fran recognized the need to learn the content relationally; however, her previous learning, the textbooks, lack of time and sometimes the instructors prevented her from doing so to her satisfaction. The frustration was especially high in Trigonometry, Probability, and the first Calculus I. She talked much about the instructors in her diary. Good instructors "presented the material in a clear manner, one

step leading to the next and explained any confusing aspects." Poor instructors gave "bits and pieces with no detailed directions as to how to fit them together," required much "short-term memory work," and did not focus on "why or how to." Basically, when Fran had a good instructor and text, she learned relationally. Otherwise, because her background would not permit her to learn content presented poorly in a relational manner, she was forced to learn enough to "get by." She wanted to understand relationally, but she could not. Her persistence and stubbornness would not allow her to quit. Despite her problems and frustrations in some of her coursework, Fran continued to teach mathematics relationally. "As a teacher, I see too many students who are interested in the correct answer only. These students generally are not able to work similar problems because they didn't take time or weren't interested in why something works. I would much rather emphasize thinking skills and make allowances for minor computational errors."

In the Fall 1985 interview, Fran's understanding remained the same. As much as possible, she tried to understand relationally the mathematics presented to her. However, her background, lack of time, and the mode of instruction often interfered--although not as much as the previous semesters. Her coursework strongly reinforced her desire to teach students relationally. "It's very

frustrating to go into a classroom and have someone put something on the board and assume that you know where every part of that particular step and procedure came from, and you don't. But I always go back in the beginning and try to make sure that they (the students) have the definitions and terms and relationships between the terms and what they stand for, and put in all the steps, even if they know how to do it. At least they have seen where I'm getting what I'm doing from."

In the final interview, Fran was asked if the program had affected her teaching of mathematics. She responded, "I try to make sure that all my students understand how to do something and when they walk into the classroom from the very beginning, we work with removing the fear of mathematics." When asked what she learned in the program, she responded, "The more I took the more I knew I didn't know. And I now know that there is a lot I still don't know. I'm not real comfortable about it. I feel like I know where to go to find the information if I need it. I know where to start anyway." Fran became a better teacher as a result of the program, but not because she learned more mathematics--actually she learned that she did not know very much. But her experiences helped her better understand her students' frustrations and what they needed from a teacher to learn and enjoy mathematics. If the end justifies the means, the program was a success for Fran.

Similarities and Differences Among Pam, Tina and Fran

Several noteable similarities and differences among the three teachers with respect to their affect, conceptions and understanding require further mention.

Affect. Pam and Fran entered the program with positive attitudes toward mathematics; Tina's attitude was neutral at best. By the end of the program, attitudes remained or became positive. However, anxiety toward mathematics--especially in testing situations-- increased as a result of the program. The pace and intensity of the program seemed to be a primary contributor. Tina and Fran were driven especially by a personal desire to perform well and prove themselves. They were determined to conquer rather than be conquered. The teachers experienced much frustration during the program. Pam and Fran were frustrated because they could not understand the mathematics; Tina was frustrated by the pace and her test performance. Finally, none of the teachers left the program confident in their ability to do and understand college mathematics; however, Tina and Fran became more confident in their ability to teach mathematics.

Conception. Coincidently, each teacher entered the program at a different stage of Perry's (1970) scheme with respect to conceptions of mathematics. Tina was dualistic, Fran was multiplistic, and Pam was relativistic. Interestingly, each teacher remained at the same stage

throughout the program. The thirty semester hours of mathematics moved neither of them along Perry's scheme. This result was quite surprising.

Understanding. Although all three teachers had received good grades in their school and undergraduate mathematics courses, their previous learning of mathematics had been highly instrumental. Their background in mathematics had not adequately prepared them for the coursework of the program. Unfortunately, the pace and nature of the coursework forced them to continue learning mathematics instrumentally. Pam was aware of the problem but could do nothing about it. Tina was frustrated by her inability to memorize content and became heavily reliant on textbooks. Fran learned relationally when she had the opportunity, but could seldom do so. The teachers simply did not understand and retain much of the content of the program.

Fran taught mathematics relationally from the outset, and the program provided her more incentive to continue the practice. On the other hand, Pam and Tina were highly intrumental in their teaching prior to the program. However, some evidence of relational teaching was found in both by the end of the program. Whether the change in teaching was due to the program, experience, or change in teaching assignments could not be ascertained.

Implications for Further Research

Before discussing implications for further research, a word of caution against over-generalizing the results of this study is in order. Each teacher entered the certification program to retain a current teaching position. Her goal upon entering the program was to finish it. She was not necessarily driven by a love and appreciation of mathematics; she was driven by a love of teaching and a desire to retain her teaching position. These goals are not necessarily consistent with the goals of most persons seeking a secondary certificate in mathematics.

These case studies raise questions concerning the study of teachers' conceptions of mathematics. Perry's (1970) scheme provided a clear means to classify and describe the teachers' conceptions of mathematics. Creating instruments and interview questions concerning conceptions was quite easy. However, the scheme is unidimensional. We found other dimensions of conceptions of mathematics pertaining to complexity, utility, and origin which were not encompassed in Perry's scheme. Future research on teachers' conceptions of mathematics should focus on more than one dimension. Perry scheme provided useful information, but the information is quite incomplete.

This investigation illustrated a strong relationship between affect, conceptions and understanding. The teachers' affect concerning mathematics was heavily dependent upon their conception and understanding. For

example, Pam's view of mathematics was relativistic; however, she was much more comfortable in a dualistic mathematical world. Fran was frustrated in trying to learn relationally in an instrumental environment. Future research on teachers' affect must consider conceptions, belief systems, and understanding. Additionally, further research on the relationships among these constructs is desperately needed.

Finally, the success of the certification program with respect to Pam, Tina and Fran was questionable at best. The program seemed to have no effect on their conceptions of mathematics. Attitudes remained or became positive, but anxiety and frustration increased. Their learning was highly instrumental, and Pam's and Tina's teaching became slightly more relational. One would hope that a program of collegiate mathematics to prepare secondary teachers would heighten affect, raise conceptions and increase understanding. Why this program failed to accomplish the latter two goals is a question for further research. The coursework was relatively traditional, and the instructors, for the most part, were highly competent. How should such programs be changed to better influence conceptions and understanding? Were the courses and teaching practices appropriate for this group of teachers? If not, what courses and practices would be more appropriate? Further research to answer these questions is needed.

Closing Remarks

This study provided detailed descriptions of the affect, conceptions and understanding of three teachers as they proceeded through thirty semester hours of mathematics to gain a teaching certificate in secondary mathematics. Several glaring similarities among the teachers indicated that the program fell short of adequately preparing them for mathematics teaching. Hopefully, this study will stimulate further research and insights into programs designed to prepare secondary school mathematics teachers.

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TABLE 1

Certification Program Course Descriptions

Plane Trigonometry: Trigometric functions, radian measure, variation and graphs of the trigonometric functions, logarithms, solutions of right angles, trigonometric identities, trigonometric equations and inverse functions, solutions of the general triangle and complex numbers.

Analytic Geometry: A study of sets and numbers, relations and functions, inequalities; exponential, logarithmic, circular, and trigonometric functions, and elements of analytic geometry.

Calculus I: Differential and integral calculus including limits and continuity, differentiation, the mean-value theorem and applications, integration and applications.

Calculus II: A study of the logarithmic, exponential, trigonometric, and hyperbolic functions; the technique of integration; volume, work, and other applications of the integral; polar coordinates and parametric equations.

Calculus III: A study of sequences, indeterminant forms, improper integrals, infinite series, functions of several variables, double and triple integrals.

Probability: Laws of probability, random variables, mathematical expectation, Bayes Theorum; binomial, Poisson and normal distributions; elementary exploratory data analysis.

Higher Geometry: Introduction to modern approaches of the development of Euclidean geometry.

Linear Algebra: Vector spaces, linear transformation, representation of linear transformations by matrices, matrix algebra, determinants, systems of linear equations and eigenvalue problems.

Abstract Algebra: Mappings, relations, logic, semigroups, groups, Boolean algebras, directed and undirected graphs and applications.

Mathematics Concepts and Methods: Introduction to nature, history, methods, and applications of mathematics and to mathematics journals of relevance to secondary teachers.

TABLE 2

Course Schedules and Orientations of Pam, Tina and Fran

		Spring 1984	Summer 1984	Fall 1984	Spring 1985	Summer 1985	Fall 1985	Spring 1986
PAM	Courses	(Previous) College Alg. Trigonometry Analytic Geo. Calculus I Methods	College Alg. Trigonometry Analytic Geo. Probability	Calculus I (withdrew)	Calculus I	Calculus II Calculus III Geometry Abstract Alg.	Linear Alg.	
	Attitude Anxiety Conception Learning Teaching	Positive A little Relativistic Instrumental Instrumental			Positive Increased Relativistic Instrumental Instrumental		Positive Same Relativistic Instr/Rel Instr/Rel	
TINA	Courses	(Previous) College Alg. Trigonometry Analytic Geo.	Analytic Geo Probability	Calculus I	Calculus II Methods	Calculus III Geometry Linear Algebra Abstract Alg.		
	Attitude Anxiety Conception Learning Teaching	Neutral A little Dualistic Instrumental Instrumental		Positive Increased Dualistic Instrumental Instrumental	Positive Decreased Dualistic Instrumental Instrumental		Positive Decreased Dualistic Instrumental Instr/Rel	
FRAN	Courses	(Previous) College Alg.	Trigonometry Analytic Geo. Probability	Calculus I (withdrew)	Calculus I Methods	Calculus II Calculus III Abstract Alg.	Linear Alg.	
	Attitude Anxiety Conception Learning Teaching	Highly Positive None Multiplistic Instrumental Relational			Positive Increase Multiplistic Instr/Rel Relational		Positive Same Multiplistic Instr/Rel Relational	Positive Same Multiplistic Instr/Rel Relational